Constant Jordan Type modules of Loewy Length 2

Daniel Bissinger

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- For $r \in \mathbb{N}$ we denote by $E_r \cong (\mathbb{Z}/(p))^r$ an elementary abelian p-group of rank r.

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$\rightsquigarrow$ full subcategories $\mathrm{CJT}(kE_r)$, $\mathrm{EIP}(kE_r)$ and $\mathrm{EKP}(kE_r)$.

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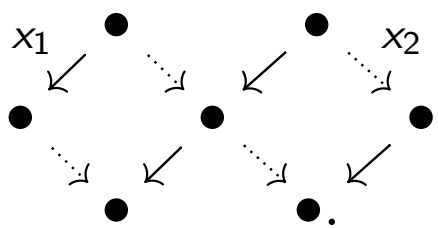
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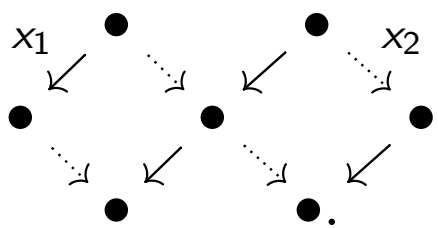


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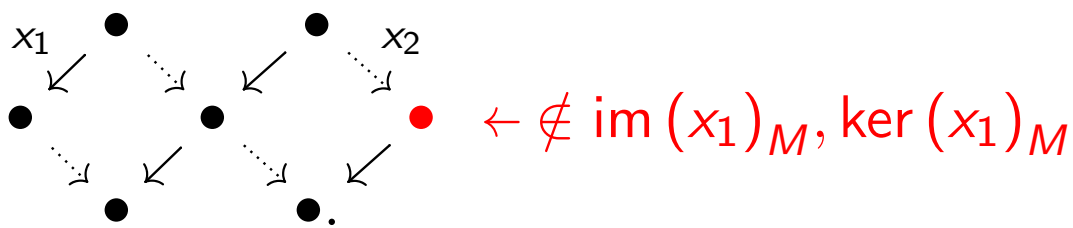
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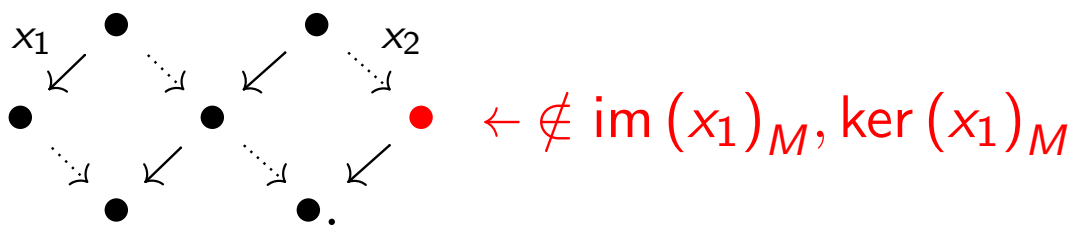
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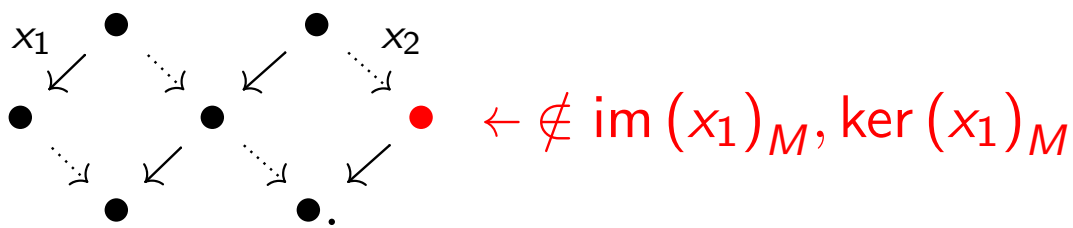
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Question: Which (stable) Jordan types are realized as the (stable) Jordan type of (indecomposable) modules in CJT(kE_r)?

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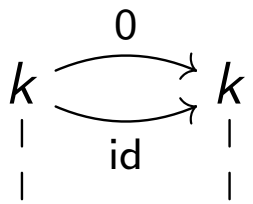
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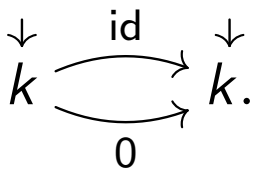
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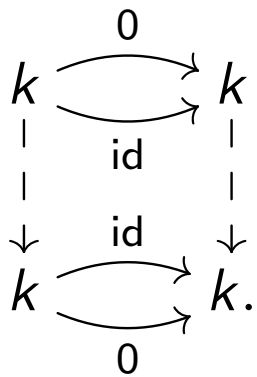
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We have an exact, faithful, dense functor $\mathfrak{F}\colon \operatorname{rep}(\Gamma_r) \to \operatorname{mod}_2 kE_r$

$$\mathfrak{F}(M) := M_1 \oplus M_2 \text{ and } x_j.(m_1 + m_2) = M(\gamma_j)(m_1)$$

for all $m_1 + m_2 \in \mathfrak{F}(M)$, $1 \leq j \leq r$.

- $\mathfrak{F}$ almost reflects isomorphism classes on indecomposables: $\mathfrak{F}(S_1) = k = \mathfrak{F}(S_2)$, where $S_1 := k \xrightarrow{r} 0$, $S_2 := 0 \xrightarrow{r} k$.
- $\mathfrak{F}$ is far from being full ($r \geq 2$):

$$M := \quad k \overset{0}{\underset{\text{id}}{\rightrightarrows}} k$$

$$N := \quad k \overset{\text{id}}{\underset{0}{\rightrightarrows}} k.$$

$\Rightarrow \operatorname{Hom}(M, N) = 0 = \operatorname{Hom}(N, M)$, but $\operatorname{Hom}_{kE_r}(X, Y) \neq 0$ for all $X, Y \neq 0$ as kE_r is local.

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$\rightsquigarrow$ Study $\mathrm{CJT}_2(kE_r), \mathrm{EKP}_2(kE_r)$, $\mathrm{EIP}_2(kE_r)$ with new tools in $\mathrm{rep}(\Gamma_r)$.

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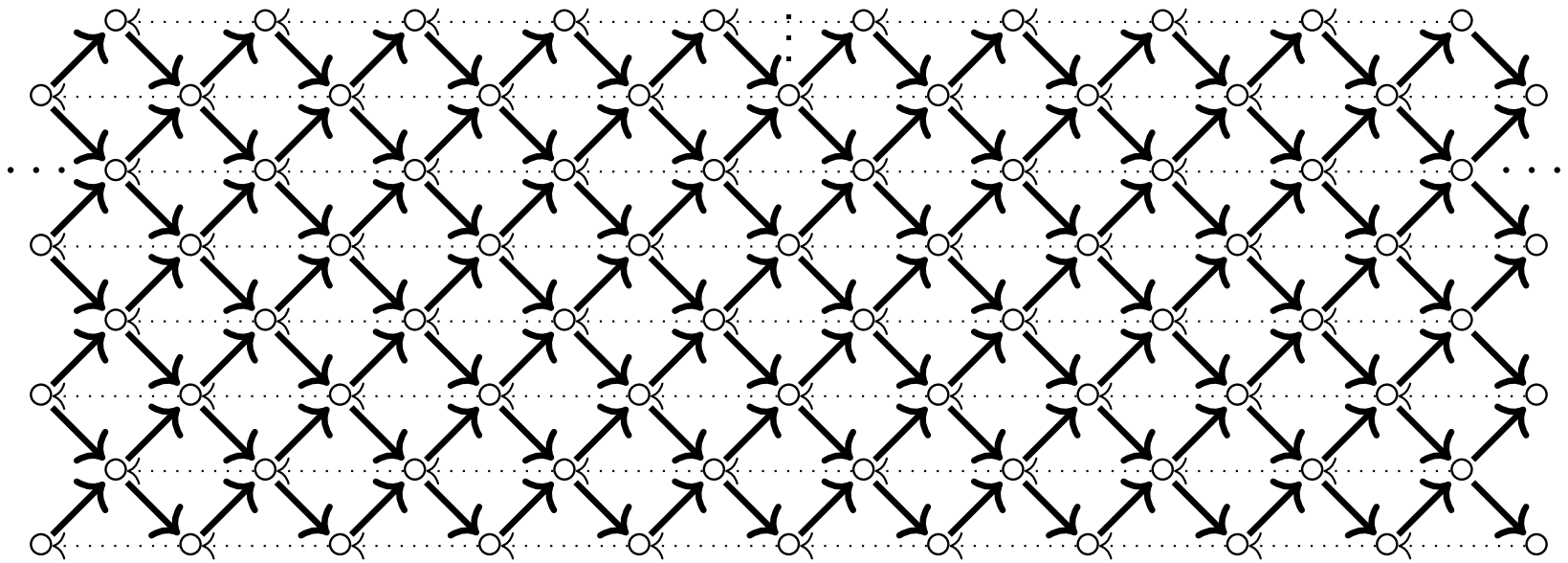
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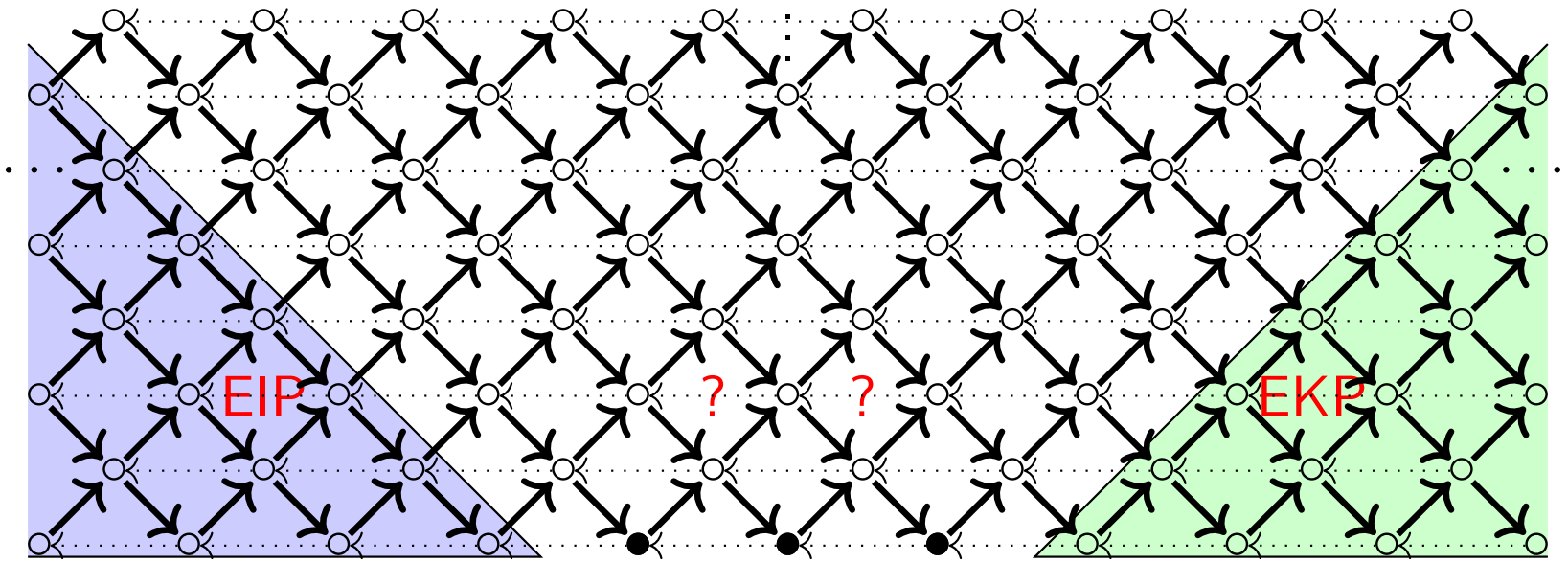


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- $M \in \mathcal{P}, \mathcal{I}$, then $M \in \mathrm{EKP}(\Gamma_r) \cup \mathrm{EIP}(\Gamma_r)$. Moreover, M and the Jordan type of $\mathfrak{F}(M)$ can be computed.

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Examples:

- $k \underset{\mathrm{id}}{\overset{0}{\rightrightarrows}} k$ has generic Jordan type $(0, 1)$.
- Let $M \in \mathrm{CJT}(\Gamma_r)$, then $\mathfrak{F}(M)$ has constant Jordan type $[1]^{c_M}[2]^{d_M}$.

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Example (of an unuseful filtration)

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Let $0 \to A \to M \to B \to 0$ be a short exact sequence, then

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$\rightsquigarrow$ Study the generic Jordan type of elementary representations.

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We say that a positive root has the **equal kernels property**, provided every indecomposable representation with dimension vector δ is in $\mathrm{EKP}(\Gamma_r)$.

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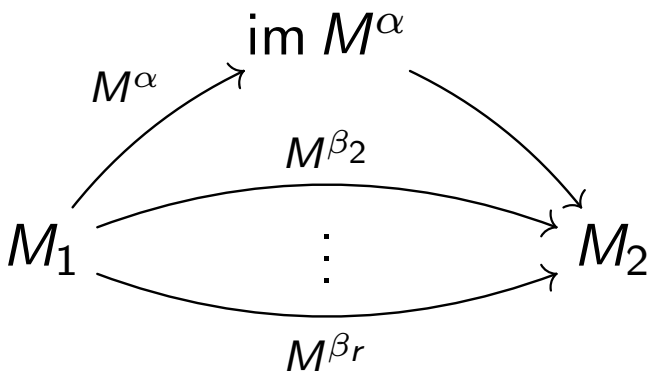
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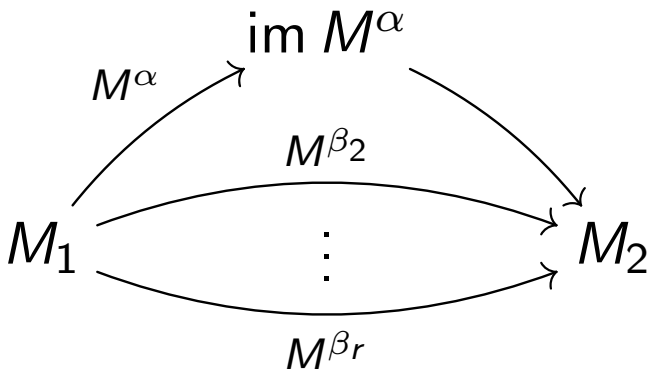
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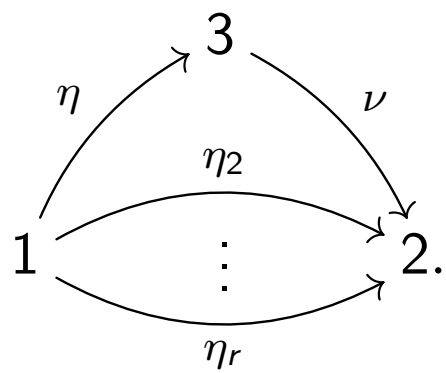


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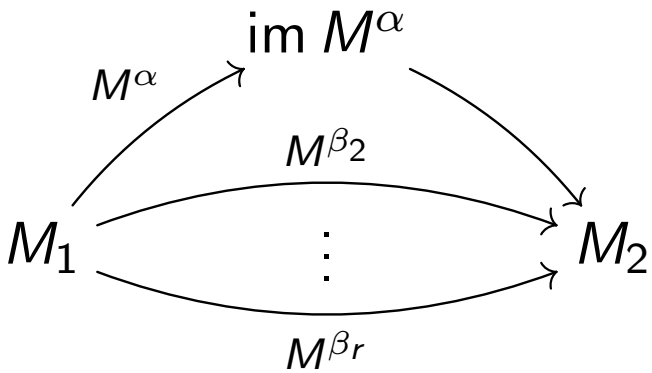


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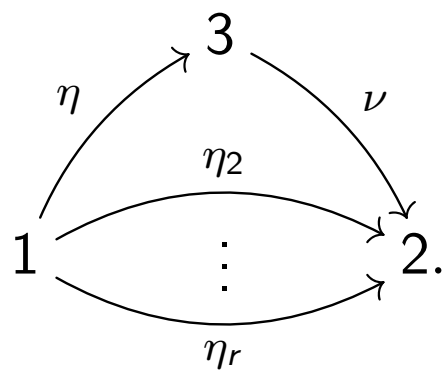


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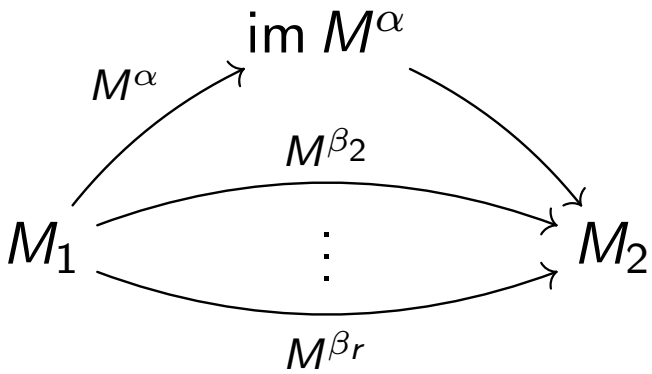
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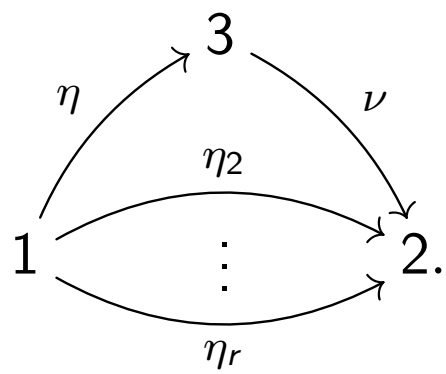
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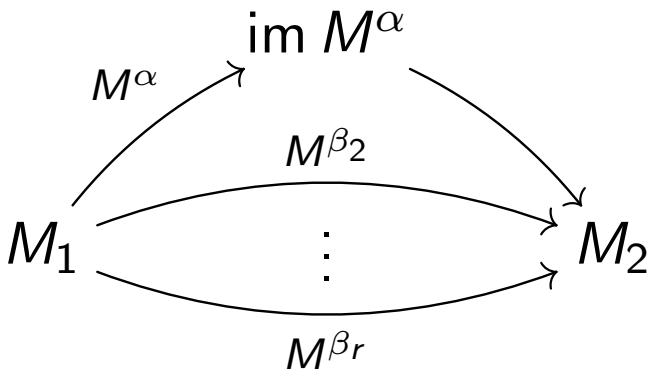
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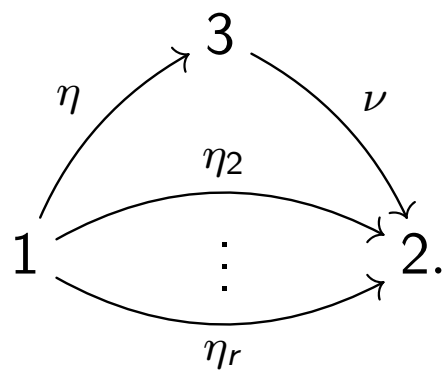
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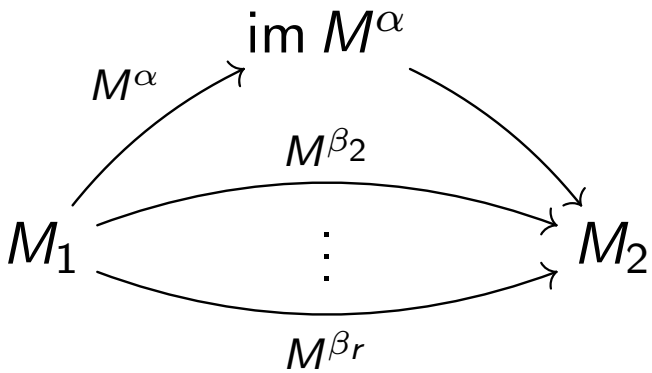


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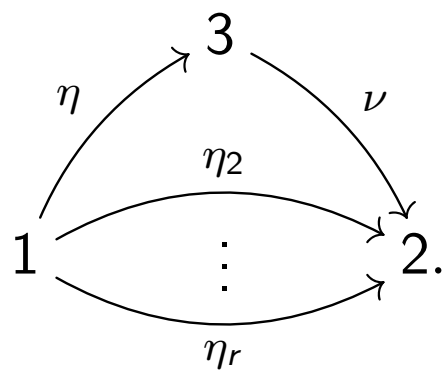
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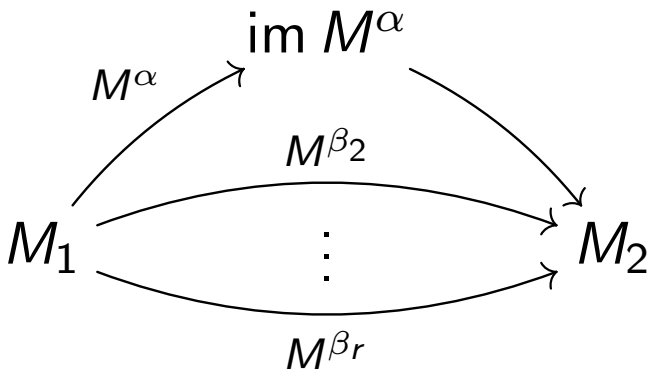


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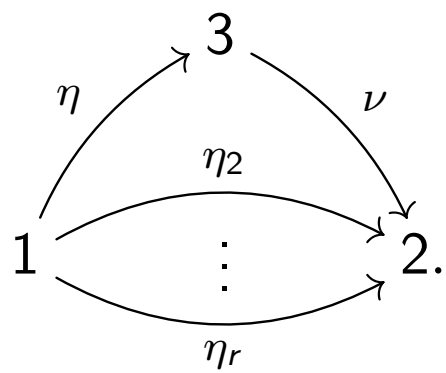
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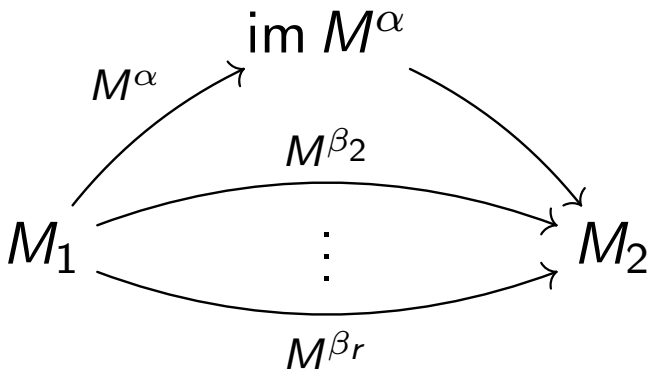
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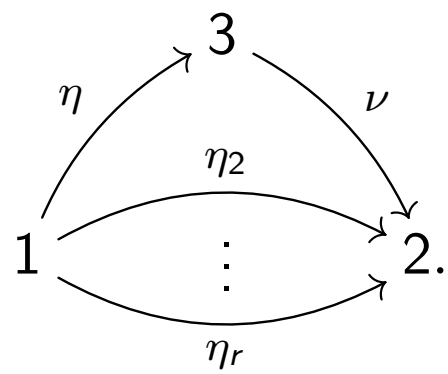
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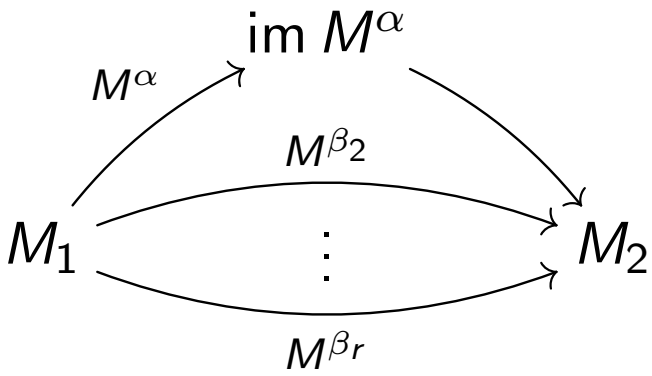
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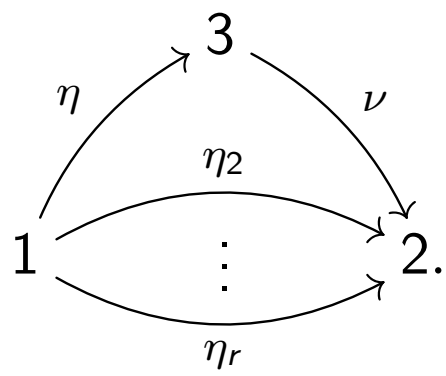
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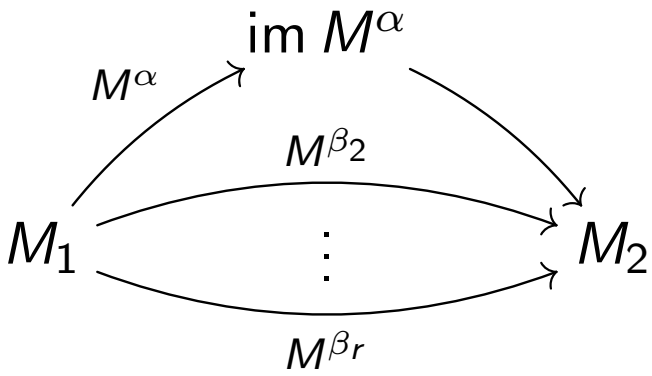
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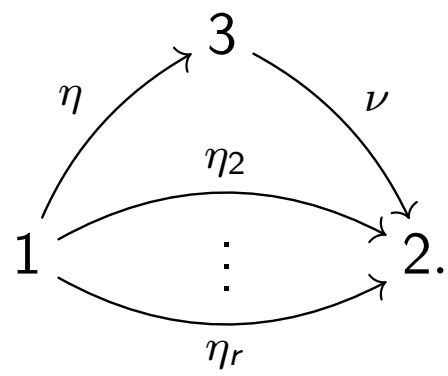
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